1.4 Proving Conjectures: Deductive Reasoning

Curricular Competencies

I can use play, inquiry and problem solving to gain understanding
I can explain and justify math ideas and decisions
I can apply flexible and strategic approaches to problems

Recall: Inductive Reasoning: drawing a conclusion by observing patterns and identifying properties in specific examples

Deductive Reasoning: drawing a specific conclusion through logical reasoning by starting with general assumptions

Ex. Make a deduction in each of the following cases.

TRANSITIVE PROPERTY

$A \to B$, $B \to C$ then $A \to C$

a. Paul lives in Medicine Hat. Medicine Hat is in Alberta.
   Paul lives in Alberta
b. Every animal has a heart. All dogs are animals.
   All dogs have hearts
c. The sum of any two consecutive whole numbers is an odd number. The whole numbers 11 and 12 are consecutive.
   The sum of 11 and 12 is odd.
d. The diagonals of a parallelogram bisect each other. *PQRS* is a parallelogram.
   The diagonals of PQRS will bisect each other
e. The diagonals of a rhombus intersect at right angles. *KLMN* is a rhombus.
   The diagonals of KLMN intersect at right angles

Proof: mathematical argument showing that a statement is valid in all cases, no counterexample exists

We can use DEDUCTIVE reasoning to PROVE a statement is true.

Remember: We can use inductive reasoning to make conjectures and find evidence to support our conjecture. We may be able to find a counterexample to prove our conjecture is false but we can NEVER prove a conjecture is true.

Ex 2: Example 2: Jon discovered a pattern when adding integers:

$$1 + 2 + 3 + 4 + 5 = 15$$
$$(-15) + (-14) + (-13) + (-12) + (-11) = -65$$
$$(-3) + (-2) + (-1) + 0 + 1 = -5$$

He claims that whenever you add five consecutive integers, the sum is always 5 times the median of the numbers. Prove Jon's conjecture using an algebraic method.

↳ middle number

Let x be the median value

$x-2, x-1, x, x+1, x+2$ ← general case for 5 consecutive

$(x-2)+(x-1)+x+(x+1)+(x+2)$

$5x+0$

$\underbrace{5x}_{\text{5 times the median value}}$

Ex 3: Prove that the sum of any two odd numbers is an even number.

$7+13=20$

$11+3=14$

To represent even #s

$2x$

To represent odd #s

$2x+1$

PROOF:

$(2x+1)+(2y+1)$

$2x+2y+2$ Factor out a 2

$2(x+y+1)$

(since 2 is a factor this is always even

Ex 5: The following is an example of a number trick. Use inductive reasoning and three trials to determine the answer each time.

Choose a number. Double it. Add 5. Add your original number. Add 7. Divide by 3. Subtract your original number.

	1	2	3
Original Number			
Double			
Add 5			
Add Original #			
Add 7			
Divide by 3			
Subtract Original #	4	4	4

Prove deductively what the answer should be each time.

let x be original #

original x

double $2x$

add 5 $2x+5$

add original # $2x+5+x$

$3x+5$

add 7 $3x+12$

divide by 3 $\frac{3x+12}{3} = x+4$

subtract original $x+4-x = 4$

A two-column proof is used in mathematics to prove a statement is true. One of the columns contains __facts__ that are known to be true and the other column contains __evidence__ describing why the corresponding statement is true. (refer to p 29 ex 4)

Practice: pg 31 #1, 2, 4, 5, 7, 8, 10, 15

2, 7, 8