1.5: Proofs That Are Not Valid

Curricular Competencies

I can solve problems with persistence and a positive attitude
I can explain and justify math ideas and decisions
I can use mistakes as opportunities to advance learning

Invalid proof A proof that contains an error in reasoning or that contains invalid assumptions

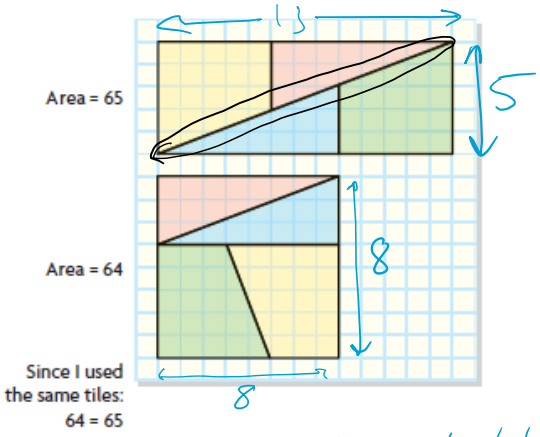
Circular reasoning An argument that is incorrect because it makes use of the conclusion to be proven.

Consider the following statement: There are tthree errorss in this sentence. Is the statement valid? Explain.

There are 2 errors which makes the statement invalid but that means another error. It is an invalid, otherwise it is both true and false at the same time.

This is an example of circular reasoning.

Ex: The following is a graphical attempt to show that $64 = 65$.



Since we know that $64 \neq 65$ not true, what is wrong with the argument?

The bold black line that outlines the shapes in the rectangle covers empty space

does not state that all students attend French

Ex. Students at DPTS attend French classes. Jack is a student at DPTS. Therefore Jack attends French class. What is the error in the reasoning?

invalid assumption, not all students have French.

Ex: Bev claims she can prove that $3=4$. She starts with 3 variables such that $a+b=c$ and then performs the following steps. Explain what she is doing at each step and identify where she went wrong.

$$4a-3a+4b-3b=4c-3c$$ expanding the equation ($a=4a-3a$)

$$4a+4b-4c=3a+3b-3c$$ rearranged equation by using opposite operations

$$\frac{4(a+b-c)}{(a+b-c)}=\frac{3(a+b-c)}{(a+b-c)}$$ factored out a 4 on left and a 3 on the right; dividing by (a+b-c) to get rid of it.

$$4=3$$

$\frac{8}{2}=\frac{2x}{2}$

Bev divided by (a+b-c) which equal to zero. Not allowed.

$a+b=c$

$-c \quad -c$

$a+b-c=0$

Ex: Hossai is trying to prove the following number trick:

> *Choose any number. Add 3. Double it. Add 4. Divide by 2. Take away the number you started with.*

Each time Hossai tries the trick, she ends up with 5. Her proof, however, does not give the same result.

Hossai's Proof

n	Choose any number.
$n+3$	Add 3.
$2n+6$	Double it.
$2n+10$	Add 4.
$2n+5$	~~Divide by 2.~~
$n+5$	Take away the number you started with.

$2(n+3)$

$2n+6$

$\frac{2n+10}{2}$

$\frac{2n}{2}+\frac{10}{2}$

$n+5$

$-n$

5

Where is the error in Hossai's proof?

Hossai didn't divide by 2 properly.

In Summary:

- A single _____ in reasoning will break down the logical argument of a deductive proof. This will results in an _____ conclusion, or a conclusion that is not _____ by the proof.
- Division by _____ always creates an error in a proof.
- _____ reasoning must be avoided! Be careful not to assume a result that follows from _____.
- The reason you write a proof is so that others can _____ and _____ it. After you write a proof, have someone else read it; if they get _____, fix it!

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