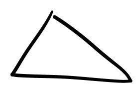
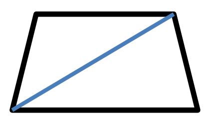
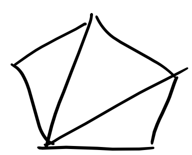
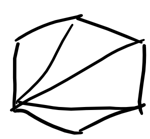
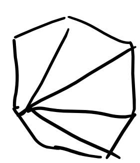
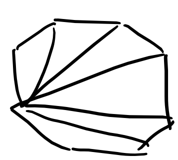
Date ________________ Foundations Math 11

# 2.4: Angle Properties in Polygons

**Investigating the INTERIOR angle sum of polygons**

Complete the table below to help you make a conjecture about the interior angle sum of any n-sided polygon.

| Polygon | Number of Sides | Sketch | Number of Triangles | Sum of Interior Angles |
|---|---|---|---|---|
| Triangle | 3 |  | 1 | 180° |
| Quadrilateral | 4 |  | 2 | 360° |
| Pentagon | 5 |  | 3 | 540° |
| Hexagon | 6 |  | 4 | 720° |
| Heptagon | 7 |  | 5 | 900° |
| Octagon | 8 |  | 6 | 1080° |
| n-gon | $n$ | ? | $n-2$ | $180(n-2)$ |

Use your conjecture to predict the interior angle sum of a dodecagon (12 sides). Verify your prediction using triangles.

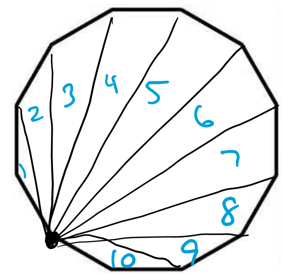
Sum of interior angles

$180(n-2)$

$180(12-2)$

$180(10)$

$1800°$



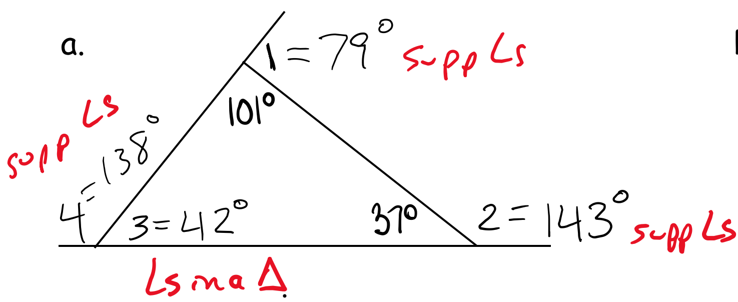

**Investigating the EXTERIOR angle sum of polygons**

Find the exterior angles in each diagram below. (not drawn to scale)

$90+60+80+85+45$

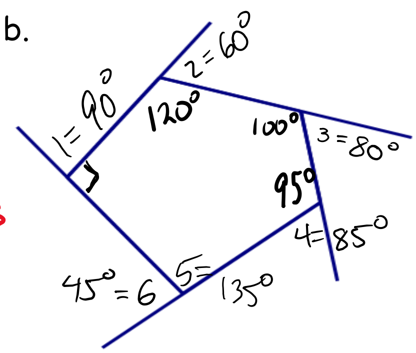
$= 360°$

a.




$138+79+143=360°$

b.




Pentagon adds to 540°

so 540
− 90
− 120
− 100
− 95
135°

Use inductive reasoning to make a conjecture about the exterior angle sum of a polygon.

Exterior angle sum of any polygon is 360°

Use deductive reasoning to prove your conjecture.

For n-sided polygon

interior angles $i_1, i_2, i_3, \ldots, i_n$

$$i_1+i_2+i_3+\ldots+i_n = 180(n-2)$$

$$(180-e_1)+(180-e_2)+(180-e_3)+\ldots+(180-e_n) = 180(n-2)$$

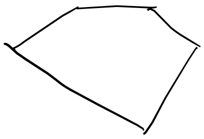
$$180n-e_1-e_2-e_3-\ldots-e_n = 180n-360$$

$$-180n \qquad\qquad -180n$$

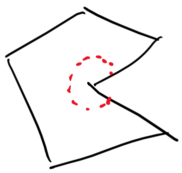
$$\frac{-1(e_1+e_2+e_3+\ldots+e_n)}{-1} = \frac{-360}{-1}$$

$$e_1+e_2+e_3+\ldots+e_n = 360$$

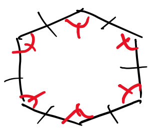
Convex polygon A polygon in which each interior angle is less than 180°



Concave polygon A polygon in which at least one interior angle is more than 180°



Regular polygon All angles and side lengths of the polygon are equal



To summarize our results from our previous investigations:

Interior angle sum of a Convex polygon:

* $180(n-2)$ *

on formula sheet

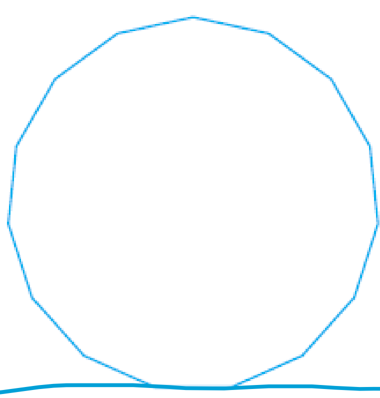
Exterior angle sum of a convex polygon:

$360°$

Interior angle of a regular convex polygon:

$$\frac{180(n-2)}{n}$$

Determine the measure of each interior angle of a regular 15 sided polygon (pentadecagon).



$$\text{interior angle} = \frac{180(n-2)}{n}$$

$$= \frac{180(15-2)}{15}$$

$$= \frac{180(13)}{15}$$

$$= 156°$$

each interior angle is 156°

P 99 1 a ~~& b~~, 2-5,8 ab, 9, 13a, 14

$$\frac{180}{180}\boxed{\frac{(180)}{182}}$$