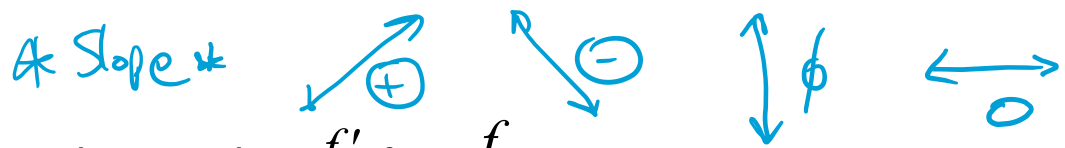
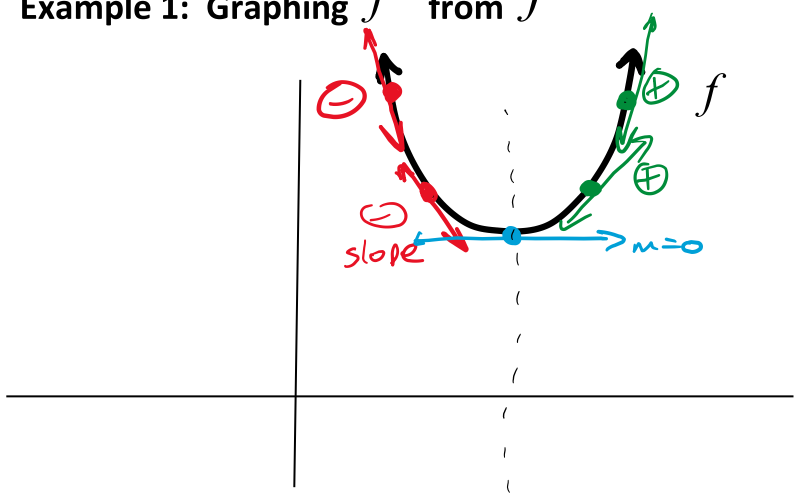
3.1.2 Derivatives, Graphs & One-Sided Derivatives

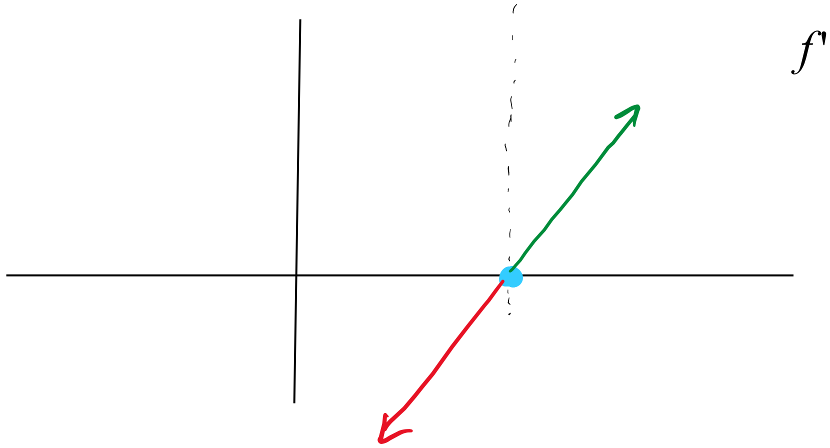
Relationship between the graphs of f – original **and** f' – derivative (slope of tangent line)



Example 1: Graphing f' **from** f



x^2
degree 2



why linear?
derivative will be degree 1

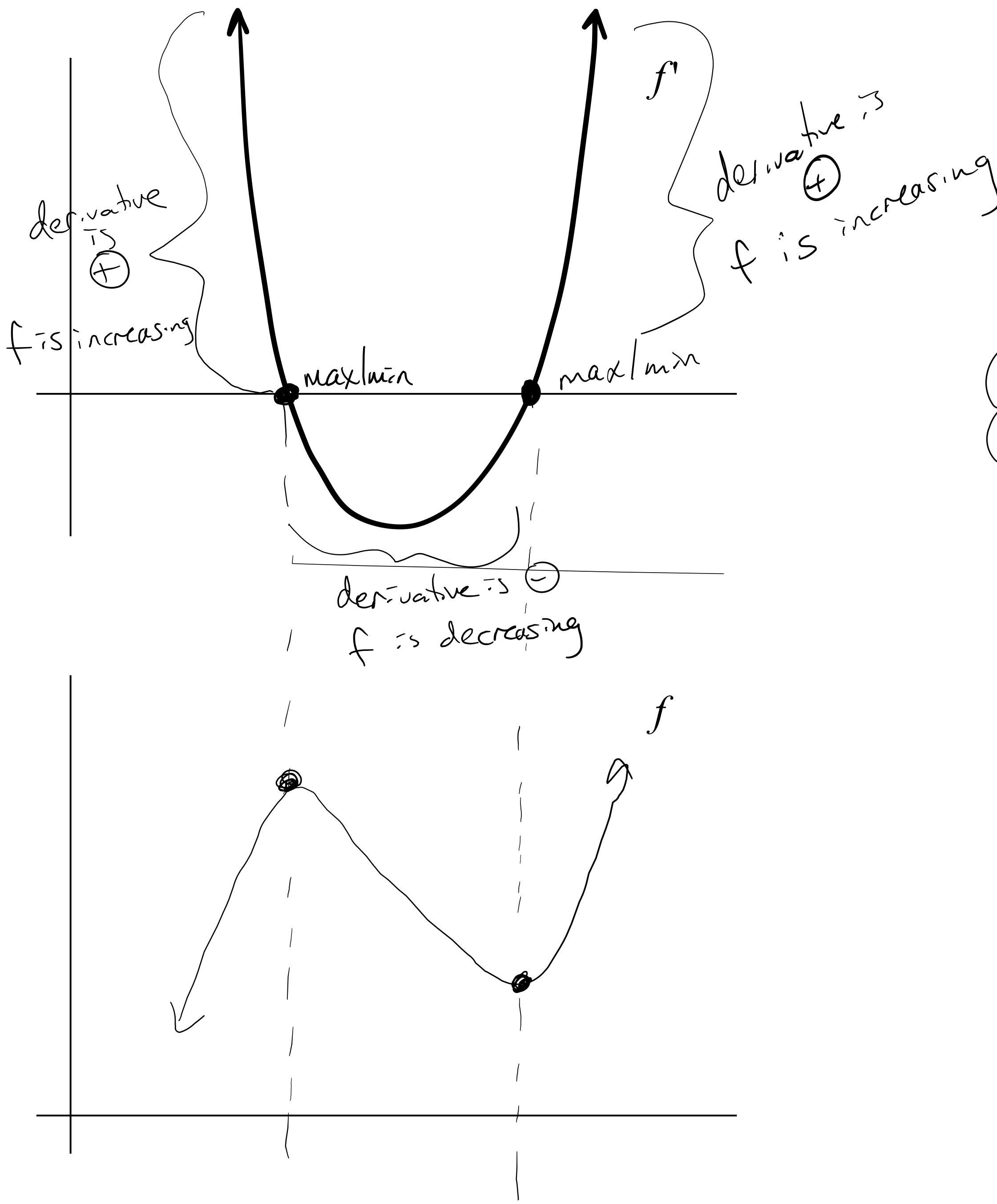
Characteristics:

f **is decreasing where** f' **is** negative

f **is increasing where** f' **is** positive

f **has a max/min where** f' **is** zero

Example 2: Graphing f from f'



x^2

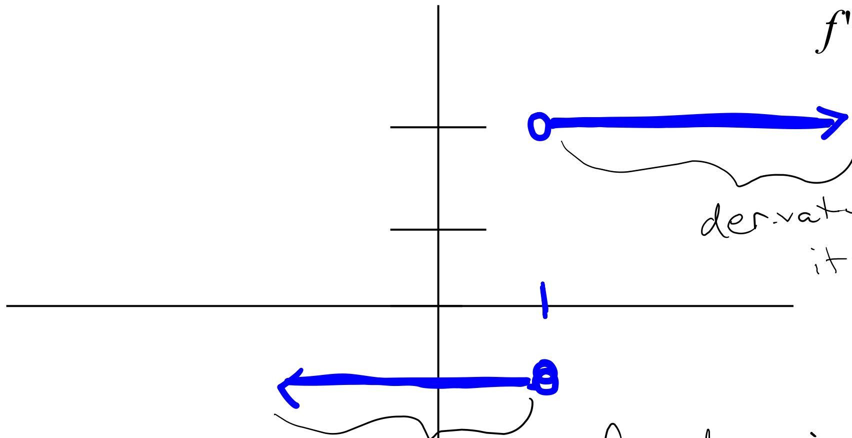
add 1 to degree for original function

?

degree 3?

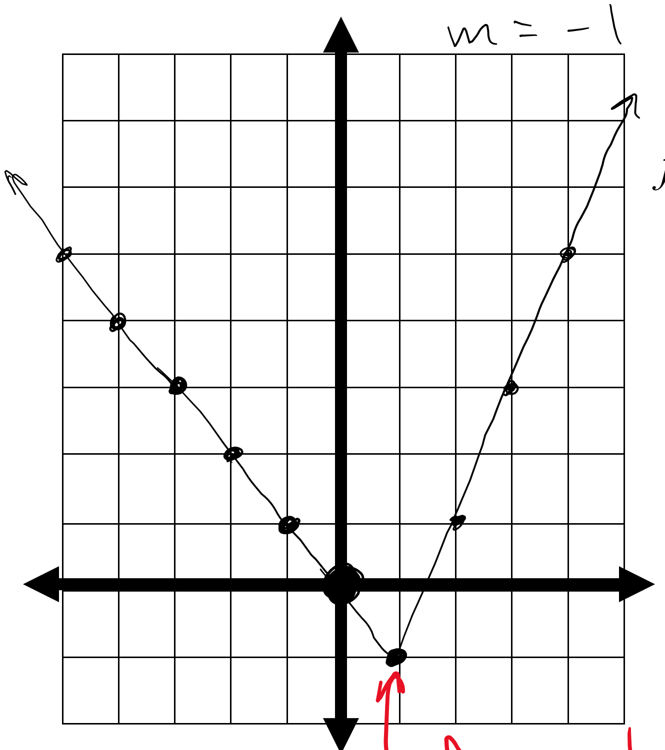
Example 3: Graph f given: $f(0)=0$, f is continuous and the graph of f' below.

(0, 0)



derivative (+) so f increasing
its constant
m = 2 for f

derivative (−) so f is decreasing
m = −1 for f



f is continuous so point is filled in even though f had open dots at x = 1

One-Sided Derivatives

The Right-hand derivative at *a* $\lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h}$

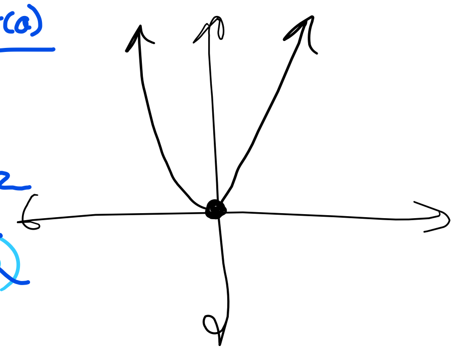
The Left-hand derivative at *a* $\lim_{h \to 0^-} \frac{f(a+h) - f(a)}{h}$

Example 4: One-Sided Derivatives Can Differ at a Point

Show that the following function has left-hand and right-hand derivatives at $x = 0$, but no derivative there.

$$f(x) = \begin{cases} x^2, & x \le 0 \\ 2x, & x > 0 \end{cases}$$

$\lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h}$



Left Hand $(a=0)$

$\lim_{h \to 0^-} \frac{(0+h)^2 - 0^2}{h}$

$\lim_{h \to 0^-} \frac{h^{\cancel{2}}}{\cancel{h}}$

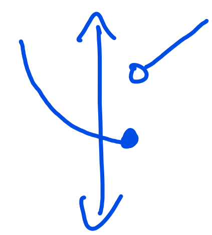
$= 0$

Right Hand

$\lim_{h \to 0^+} \frac{2(0+h) - \cancel{2(0)}}{h}$ 0^2

$\lim_{h \to 0^+} \frac{2\cancel{h}}{\cancel{h}}$

$= 2$



$0 \ne 2$ So derivative at $x=0$ DNE

Assignment 3.1.2

Page 101 – 102 # 7 – 10, 13, 14, 16-18, 22, 23, 25, 26, 28 AND:

1. Sketch a possible graph of $y = f(x)$ given the following information about its derivative.

$f'(x) > 0$ on $1 < x < 3$

$f'(x) < 0$ for $x < 1$, $x > 3$

$f'(x) = 0$ at $x = 1, x = 3$