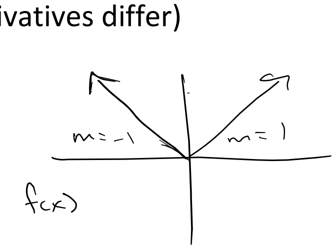
3.2 Differentiability

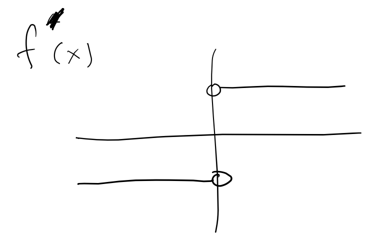
How $f'(a)$ Might Fail to Exist

A function will not have a derivative at a point $P(a, f(a))$ where the slopes of the secant lines, $\dfrac{f(x)-f(a)}{x-a}$ fail to approach a limit as *x* approaches *a*. Graphically, this would look like one of the following:

1. A *corner* (where the one-sides derivatives differ)

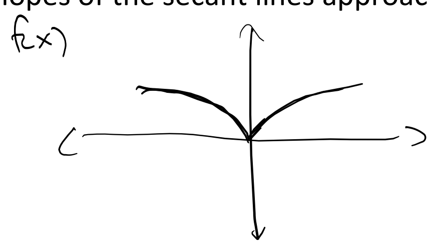
$y = |x|$

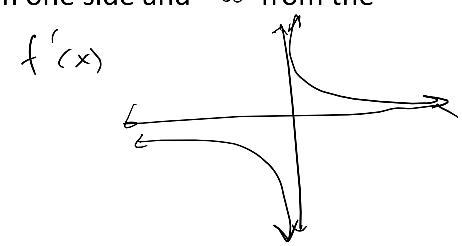




2. A *cusp* (where the slopes of the secant lines approach ∞ from one side and $-\infty$ from the other)

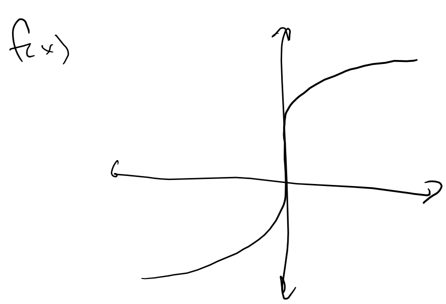
$y = x^{\frac{2}{3}}$

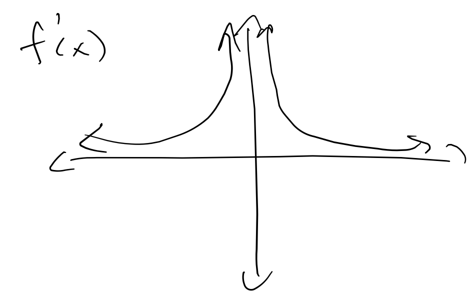




3. A *vertical tangent* (where the slopes of the secant lines approach ∞ or $-\infty$ from both sides)

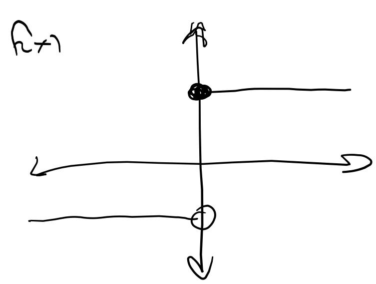
$y = x^{\frac{1}{3}}$

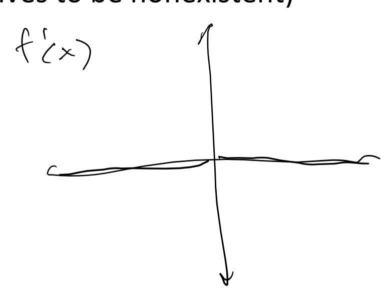




4. A *discontinuity* (which will cause one or both of the one-sided derivatives to be nonexistent)

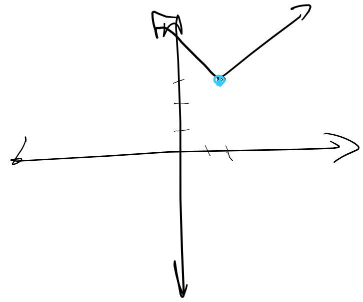
$$y = \begin{cases} -1, & x < 0 \\ 1, & x \geq 0 \end{cases}$$





Example 1: Finding Where a Function is Not Differentiable

Find all points in the domain of $f(x) = |x-2| + 3$ where f is not differentiable.



domain $f(x) \Rightarrow x \in \mathbb{R}$

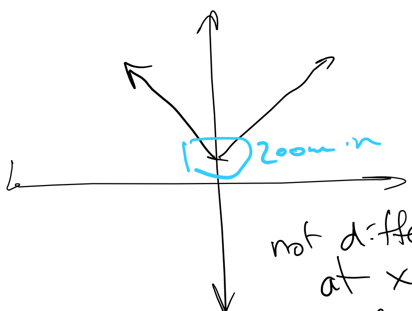
not differentiable at $x = 2$ (corner)

Most of the functions we deal with ARE differentiable including: polynomials, rationals, trigonometric, logarithmic, & exponential.

Differentiability Implies Local Linearity

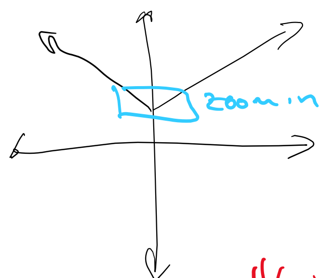
Zooming in to "See" Differentiability

$f(x) = |x| + 1$



not differentiable at $x = 0$ (corner)

$g(x) = \sqrt{x^2 + 0.0001} + .99$



differentiable everywhere on its domain

"looks like a corner but is not"

Differential functions are locally linear which means that at some point "a" the function resembles its own tangent.

Derivatives on a Calculator

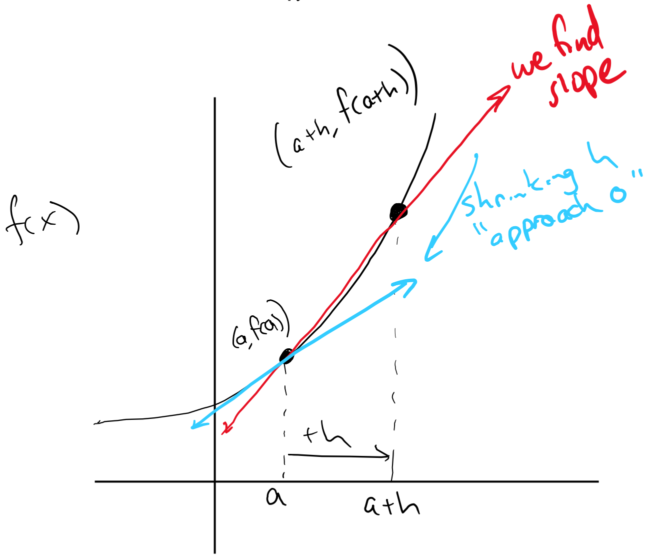
The Difference Quotient

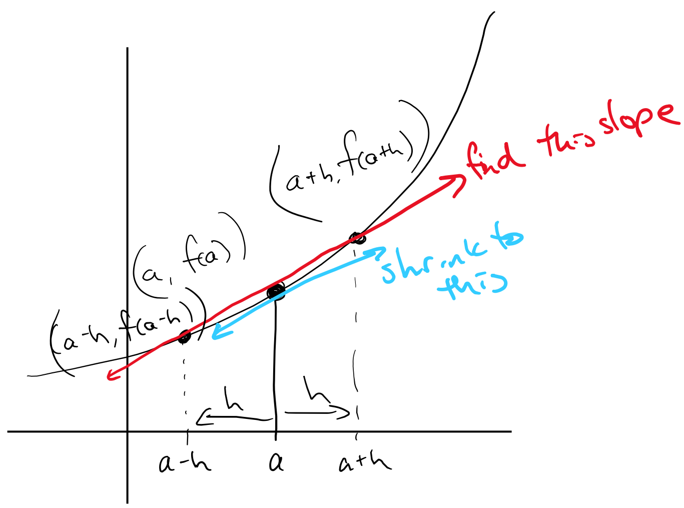
$$\frac{f(a+h)-f(a)}{h}$$

The Symmetric Difference Quotient

$$\frac{f(a+h)-f(a-h)}{2h}$$

* Calculator uses *





These formulas can be used to APPROXIMATE the derivative. As *h* becomes very small, the approximation will be more accurate. In fact, $\lim\limits_{h\to 0}\frac{f(a+h)-f(a-h)}{2h}$ has been proven to be the same as the derivative. Use each method above to calculate the numerical derivative of $f(x)=x^3$ at $x=2$ if $h=0.001$.

a

Difference

$$\frac{f(a+h)-f(a)}{h}$$

$$\frac{(2+0.001)^3-2^3}{0.001}$$

12.006001

symmetric
SDQ

$$\frac{f(a+h)-f(a-h)}{2h}$$

$$\frac{(2+0.001)^3-(2-0.001)^3}{2(0.001)}$$

12.000001

Your calculator uses the symmetric difference quotient to find the derivative.

nDeriv $\quad$ MATH $\quad$ 8: nDeriv(expression, variable, value, ___) ENTER

h=0.001 is default

nDeriv(x^3, x, 2) = 12.000001

Be careful: your calculator can be "fooled". See Example 3 page 108.

Theorem: Differentiability Implies Continuity
If f has a derivative at $x = a$, then f is continuous at $x = a$.

Be careful: Differentiability implies continuity but continuity does not necessarily imply differentiability. Why not?

Examples 1-3 on front are continuous but not differentiable.

Assignment 3.2: Page 111 – 112 #1 – 23 (use your graphing calculator to graph these functions where needed)