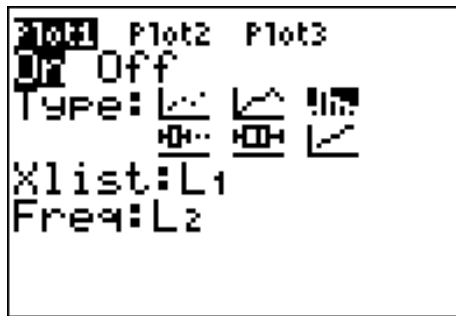# 5.4 The Normal Distribution

Investigating the Normal Curve:

Complete the frequency table below recording the results for the following task:  Roll a pair of dice 30 times and record the sum each time.
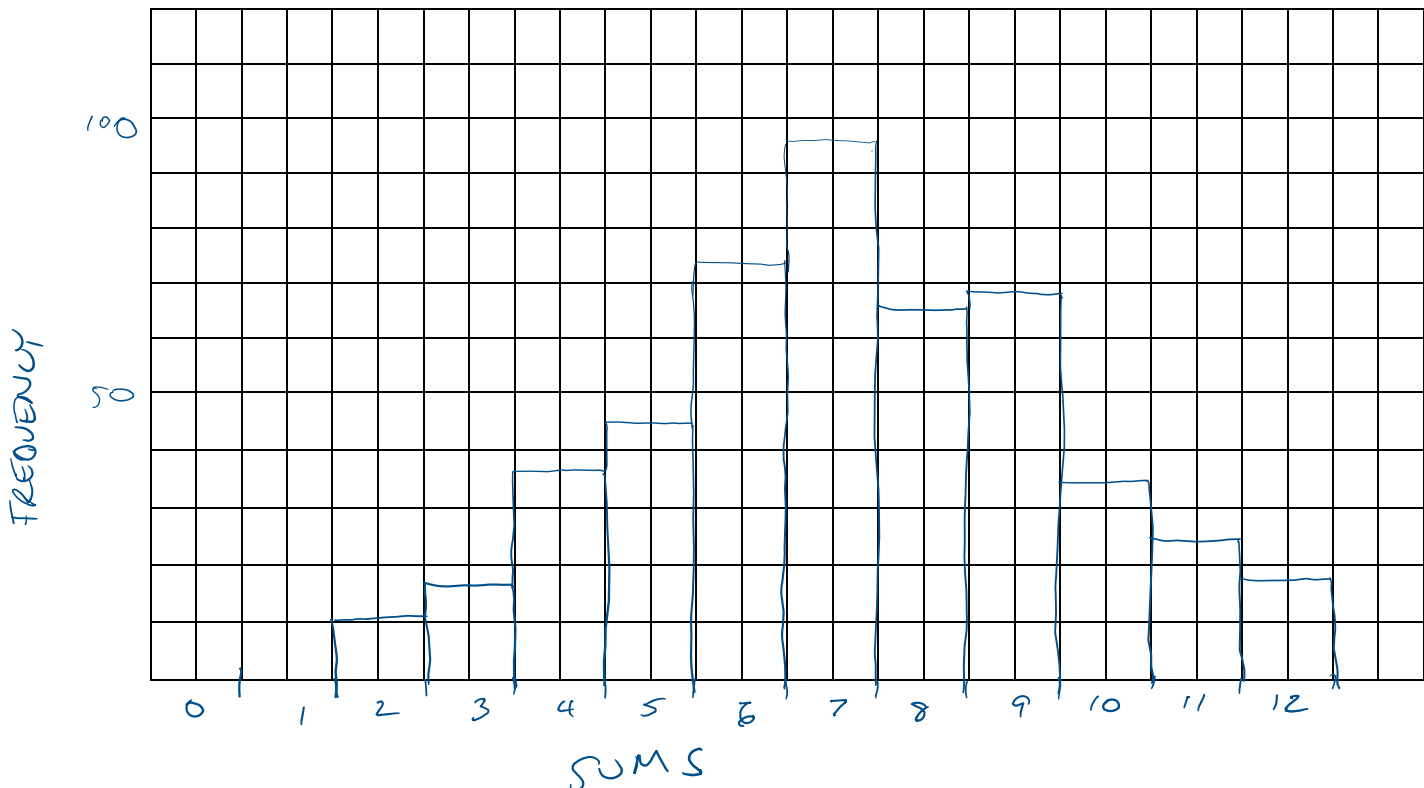
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|----|----|----|
|   |   |   |   |   |   |   |   |    |    |    |
| 12 | 18 | 38 | 49 | 74 | 97 | 67 | 69 | 35 | 25 | 18 |

Add your results to those of the rest of the class.  When all data has been accumulated, enter the sum as L1 and the frequency as L2 in your graphing calculator.  To see what the results look like:

- 2nd StatPlot
- 1
- On
- Type: histogram
- Xlist: L1
- Freq: L2
- Graph

```
Plot1  Plot2  Plot3
On  Off
Type: ⌐   ⌐   ▥
       ⊞  ⊞  ⌐
Xlist:L1
Freq:L2
```

Sketch the resulting graph below.

Normal Curve _a symmetrical curve that represents the normal distribution; also called a bell curve_

Normal Distribution _data that when graphed as a histogram or frequency polygon results in unimodal symmetric distribution_ _about the mean_

For each graph below, describe the data in terms of number of modes, distribution and what the data might possibly represent.
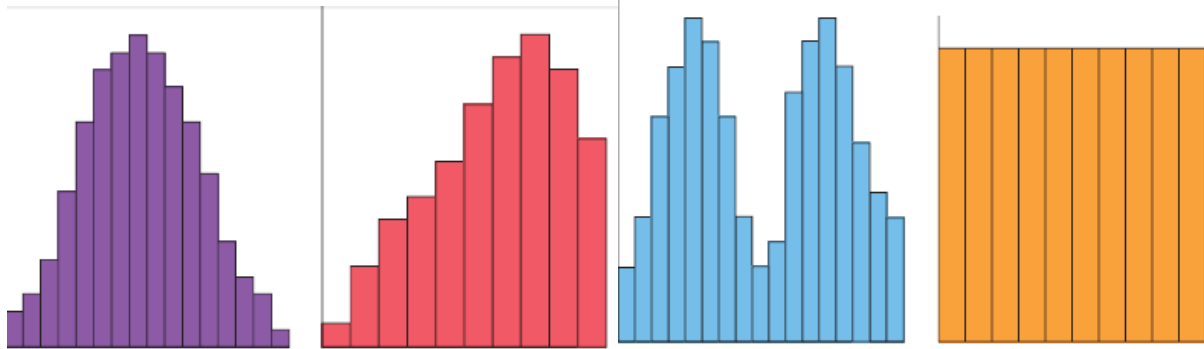


- 1 mode
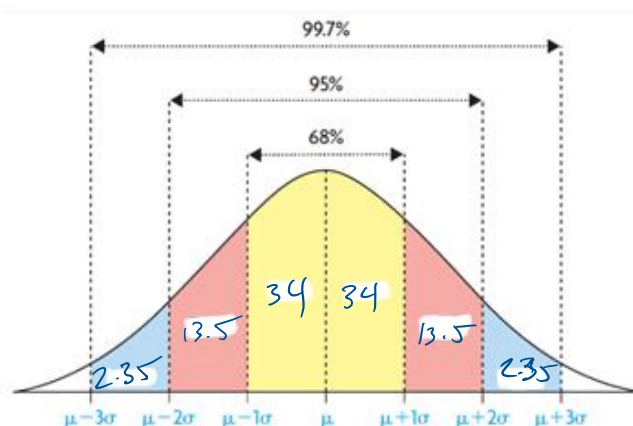- normal distribution
- temperature through the year

- 1 mode
- not normal, could be with more to the right
- heart rate, how can't be zero

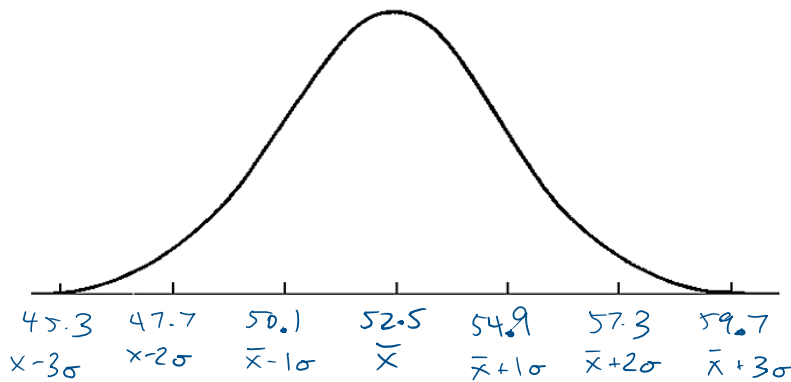- 2 modes (bimodal)
- not normal
- tree population in Australia

- all mode
- not normal
- Ryder's heart rate

What's so special about the normal distribution?

- ___68___ % of the data are within ___1___ standard deviation of the ___mean___.

- ___95___ % of the data are within ___2___ standard deviations of the ___mean___.

- ___99.7___ % of the data are within ___3___ standard deviations of the ___mean___.

Example 1:  Jim raises Siberian husky sled dogs at his kennel.  He knows, from the data he has collected over the years, that the weights of adult male dogs are normally distributed, with a mean of 52.5 lb and a standard deviation of 2.4 lb.  Complete the normal curve below, labeling the weights of the dogs along the horizontal axis.

| 45.3 | 47.7 | 50.1 | 52.5 | 54.9 | 57.3 | 59.7 |
|------|------|------|------|------|------|------|
| $x-3\sigma$ | $x-2\sigma$ | $\bar{x}-1\sigma$ | $\bar{x}$ | $\bar{x}+1\sigma$ | $\bar{x}+2\sigma$ | $\bar{x}+3\sigma$ |



Communication | **Tip**
In statistics, when an entire population is involved, use the symbol μ (read as "mu") for the mean of the population.

What percent of adult male dogs at Jim's kennel would you expect to have a weight between 47.7 lb and 54.9 lb?
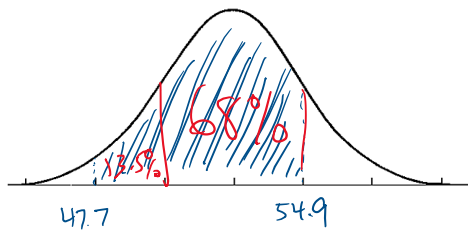
68%

47.7        54.9

68
+ 13.5
―――
81.5 %
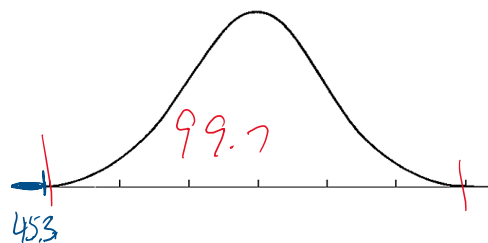
What percent of adult male dogs at Jim's kennel would you expect to have a weight between 50.1 lb and 59.7 lb?
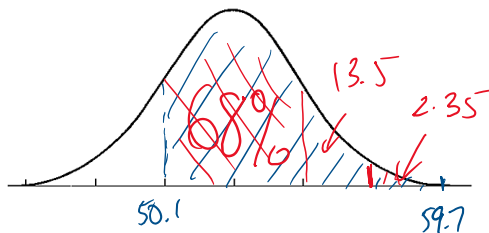
68%    13.5
             2.35

50.1        59.7

68
+ 13.5
+ 2.35
―――
83.85%

What percent of adult male dogs at Jim's kennel would you expect to have a weight less than 45.3 lb?

100
− 99.7
―――
0.3%

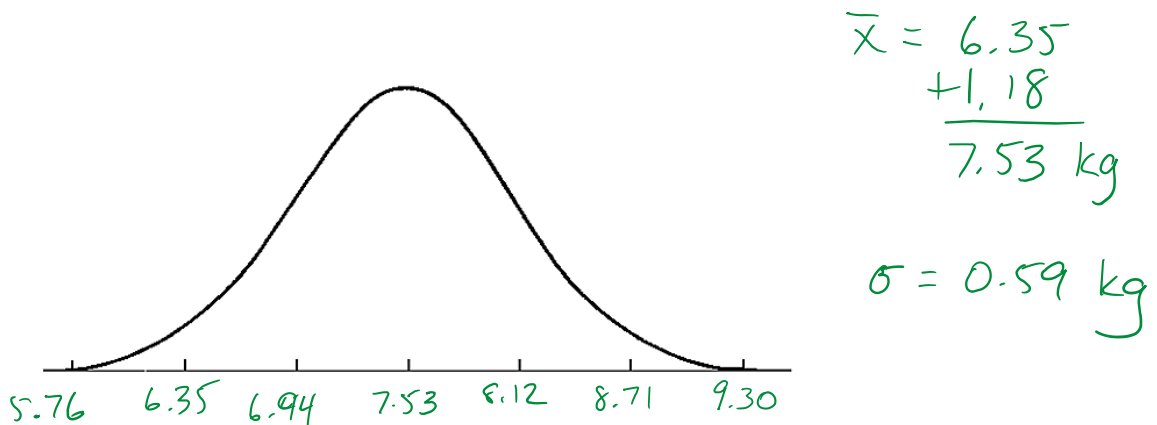$\dfrac{0.3}{2}$

0.15%

99.7

45.3

Example 2: Two baseball teams flew to the North American Indigenous Games. The members of each team had carry-on luggage for their sports equipment. The

| Team | $\mu$ (kg) | $\sigma$ (kg) |
|---|---|---|
| Men | 6.35 | 1.04 |
| Women | 6.35 | 0.59 |

masses of the luggage were normally distributed, with the characteristics shown in the table.

a) Sketch a graph to show the distribution of the masses of the luggage for each team.



| 3.23 | 4.27 | 5.31 | 6.35 | 7.39 | 8.43 | 9.47 | Men |
| 4.58 | 5.17 | 5.76 | 6.35 | 6.94 | 7.53 | 8.12 | Women |
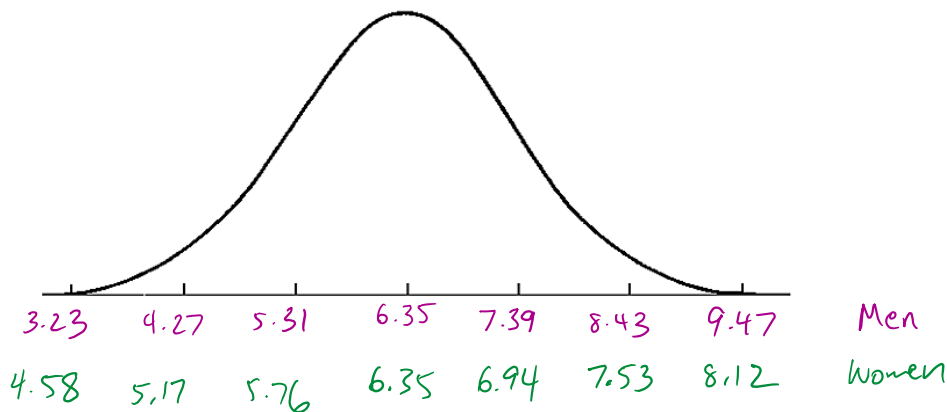
b) The women's team won the championship. Each member received a medal and a souvenir baseball, with a combined mass of 1.18 kg which they packed in their carry-on for the trip home. Sketch a graph that shows how the distribution of the masses of their carry-on luggage changed for the flight home compared to the flight there.



| 5.76 | 6.35 | 6.94 | 7.53 | 8.12 | 8.71 | 9.30 |

$$\bar{x} = 6.35$$
$$+1.18$$
$$\overline{7.53 \text{ kg}}$$
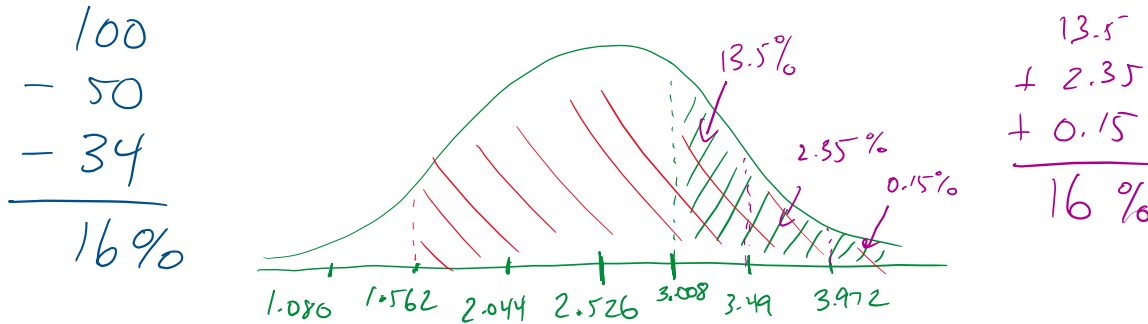
$$\sigma = 0.59 \text{ kg}$$

Does data approximate a normal distribution?

It did before, since everybody is increasing by the same amount it should again.

Prior to purchasing a new cellphone, Shirley does a lot of research. She determines that the mean number of years a cellphone lasts is 2.526, with a standard deviation of 0.482 years and a median of 2.55 years. Using this information

a) What is the likelihood that a new cellphone will last more than three years?

$$
\begin{array}{r}
100 \\
-\ 50 \\
-\ 34 \\
\hline
16\%
\end{array}
$$



13.5%

2.55%

0.15%

$$
\begin{array}{r}
13.5 \\
+\ 2.35 \\
+\ 0.15 \\
\hline
16\ \%
\end{array}
$$

1.080   1.562   2.044   2.526   3.008   3.49   3.972

b) What is the likelihood that a new cellphone will last at least 18 months?

$$
\begin{array}{r}
50 \\
+\ 34 \\
+\ 13.5 \\
\hline
97.5\ \%
\end{array}
$$

### Need to Know

- The properties of a normal distribution can be summarized as follows:
  - The graph is symmetrical. The mean, median, and mode are equal (or close) and fall at the line of symmetry.
  - The normal curve is shaped like a bell, peaking in the middle, sloping down toward the sides, and approaching zero at the extremes.
  - About 68% of the data is within one standard deviation of the mean.
  - About 95% of the data is within two standard deviations of the mean.
  - About 99.7% of the data is within three standard deviations of the mean.
  - The area under the curve can be considered as 1 unit, since it represents 100% of the data.
- Generally, measurements of living things (such as mass, height, and length) have a normal distribution.