5.5 Z-Scores

The more we analyze data, the more in-depth our analysis can become.

Hailey and Serge belong to a running club in Vancouver. Part of their training involves a 200 m sprint. Below are normally distributed times for the 200 m sprint in Vancouver and on a recent trip to Lake Louise. At higher altitudes, run times improve.

At which location was Hailey's run time better, when compared with the club results?

Location	Altitude (m)	Club Mean Time: μ for 200 m (s)	Club Standard Deviation: σ (s)	Hailey's Run Time (s)	Serge's Run Time (s)
Vancouver	4	25.75	0.62	24.95	25.45
Lake Louise	1661	25.57	0.60	24.77	26.24

Notice that Hailey's run time is less at Lake Louise but so is the club's. To make comparisons, the data must be **standardized**. We can use z-scores to accomplish this.

Standard normal distribution A normal distribution that has a mean of zero and a standard deviation of 1.

Z-Score A standardized value that indicates the # of data values above or below the mean

Z-Score Calculation Formula

$$z = \frac{x - \bar{x}}{\sigma}$$

x is the data value

$\bar{x}$ is the mean of the data set

σ is the standard deviation

Compare Hailey's Z-Scores for the two locations. Use this to determine the location where Hailey's run time was better.

Vancouver:

$$z = \frac{x - \bar{x}}{\sigma}$$

$$= \frac{24.95 - 25.75}{0.62} = -1.29$$

Lake Louise:

$$z = \frac{x - \bar{x}}{\sigma}$$

$$= \frac{24.77 - 25.57}{0.60}$$

$$= -1.33$$

Smaller number → she did better

What does it mean to have a Z-Score that is:

- Negative? lower than the mean for a given set of data
- Positive? higher " " " " " " " " "
- Zero? Same as the mean

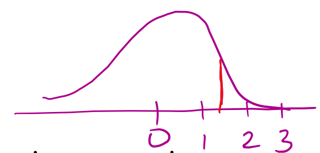
IQ tests are sometimes used to measure a person's intellectual capacity at a particular time. IQ scores are normally distributed, with a mean of 100 and a standard deviation of 15. If a person scores 119 on an IQ test, how does this score compare with the scores of the general population?

Using a Z-Score Table:

$$z = \frac{119-100}{15}$$

$$= 1.27$$

Using the Graphing Calc:



(Calculating the area under the normal curve and to the left of the Z-Score.)

2nd Vars (Distr)

2: normalcdf(

Normalcdf(start, stop, μ, σ) Enter

z	0.0	0.01	0.06	0.07
0.0	0.5000	0.5040	0.5239	0.5279
0.1	0.5398	0.5438	0.5636	0.5675
1.1	0.8643	0.8665	0.8770	0.8790
1.2	0.8849	0.8869	0.8962	**0.8980**
1.3	0.9032	0.9049	0.9131	0.9147

What does this mean?

This means that a person with an IQ of 119 is greater than 89.80% of the general population

How would an IQ score of 78 compare to the general public?

$$z = \frac{78-100}{15}$$

$$= -1.47$$

FROM TABLE

0.0708

That individual has a higher IQ than 7.08% of the general pop.

Athletes should replace their running shoes before the shoes lose their ability to absorb shock. Running shoes lose their shock-absorption after a mean distance of 640 km, with a standard deviation of 160 km. Zack wants to replace his running shoes at a distance when only 25% of people would replace their shoes. At what distance should he replace his shoes?

0.2500

Using a Z-Score Table:

z	0.09	0.08	0.07	0.06	0.05
–0.7	0.2148	0.2177	0.2206	0.2236	0.2266
–0.6	0.2451	0.2483	0.2514	0.2546	0.2578
–0.5	0.2776	0.2810	0.2843	0.2877	0.2912

$z = \frac{x - \bar{x}}{\sigma}$

$160 \cdot -0.67 = \frac{x - 640}{160} \cdot 160$

$640 + -107.2 = x - 640 + 640$

$x = 532.8$ km

Using the Graphing Calc: (Finding a score)

2nd Vars (Distr)

3: invnorm(

as a decimal

invnorm(percent below, μ, σ) Enter

The ABC Company produces bungee cords. When the manufacturing process is running well, the lengths of the bungee cords produced are normally distributed, with a mean of 45.2 cm and a standard deviation of 1.3 cm. Bungee cords that are shorter than 42.0 cm or longer than 48.0 cm are rejected by the quality control workers. If 20 000 bungee cords are manufactured each day, how many would you expect the quality control workers to reject?

Using Tables:

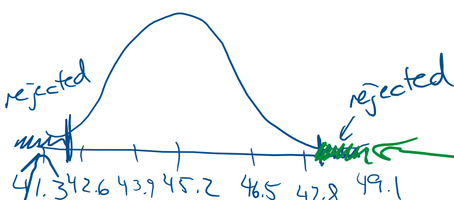
$z = \frac{42 - 45.2}{1.3}$

$z = -2.46$

from table

0.0069 or

0.69%



$z = \frac{48 - 45.2}{1.3}$

$= 2.15$

from table

0.9842 or 98.42%

100
− 98.42
1.58%

1.58
+0.69
2.27 %

20 000 × 0.0227 = 454 cords rejected

Using Graphing Calculator:

normalcdf (start, stop, $\bar{x}$, σ)

normalcdf (42, 48, 45.2, 1.3)

0.9775

97.75%

A client has placed an order for 12 000 bungee cords but will only accept bungee cords that are between 44.0 cm and 46.0 cm in length. Can this client's order be filled by one day's productions, with the equipment operating as is? Explain.

$z = \frac{44 - 45.2}{1.3}$

$= -0.92$

from table

0.1788

17.88 %

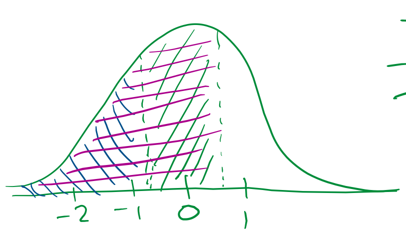
$z = \frac{46 - 45.2}{1.3}$

$= 0.62$

from table

0.7324

73.24%



73.24
−17.88
55.36 %

are acceptable

in one day

20 000
× 0.5536
11 072

bungee cords are acceptable

Need two days

Need to Know:

- Use normalcdf(...) if finding zscore (%) from a given data value
- Use invnorm(...) if finding data value from z-score

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