5.6 Confidence Intervals

We use random samples to provide information about a population when it is not possible or practical to survey an entire population. Today we look at how confident we are that our sample results actually represent the population.

Margin of Error: potential difference between a value determined from a sample and that of the true value for the population

Confidence interval: interval in which the true value you are trying to determine is estimated to be

Confidence level: the likelihood that the result for the "true" population lies within the range of the confidence interval

A telephone survey of 600 randomly selected people was conducted in an urban area. The survey determined that 76% of people, from 18 to 34 years of age, have a social networking account. The results are accurate within plus or minus 4 percent points, 19 times out of 20. How can this be interpreted if the total population of 18 – 34 year olds is 92 500?

In the social networking example:

Margin of Error: $\pm 4\%$

Confidence Interval: $76\% \pm 4\%$

Confidence Level: $\frac{19}{20} = \boxed{95\%}$

Based on the survey, what is the range of 18-34 year olds who do ~~not~~ have a social networking account?

$76 - 4\%$

$76 + 4\%$

$72\% - 80\%$

Does Sample Size Matter?

The larger the sample size, the more likely it accurately represents a population. As a result, the margin of error decreases as the sample size increases. The smaller the margin of error, the smaller the confidence interval.

Given 58% of D. P. Todd students reported loving Rolo Ice Cream (with a margin of error of $\pm$2.8%), calculate the confidence interval.

58 − 2.8

58 + 2.8

55.2% − 60.8%

Range is 60.8 − 55.2 = 5.6%

How would this interval size change if the margin of error was doubled?

58 − 5.6

58 + 5.6

52.4% − 63.6%

Range is 11.2% which doubled as well

Confidence Intervals and Sample Size:

To meet regulation standards, baseballs must have a mass from 142.0 g to 149.0 g. A manufacturing company has set its production equipment to create baseballs that have a mean mass of 145.0 g.

To ensure that the production of equipment continues to operate as expected, the quality control engineer takes a random sample of baseballs each day and measures their mass to determine the mean mass. If the mean mass of the random sample is 144.7 g to 145.3 g, then the production equipment is running correctly. If the mean mass of the sample is outside the acceptable level, the production equipment is shut down and adjusted. The quality control engineer refers to the chart below.

Confidence Level	Sample Size Needed
99%	110
95%	65
90%	45

a) What is the confidence interval and margin of error the engineer is using for quality control tests?

144.7g − 145.3g

145 g $\underbrace{\boxed{\pm 0.3\text{ g}}}_{\text{margin of error}}$

b) What does the table mean?

More samples that are tested the more confidence the engineer has in the equipment.

c) What is the relationship between sample size and confidence level?

As sample size increases the more confident we can be in our value.

A poll was conducted to ask voters who they would vote for in an upcoming election. The results indicated that 53% would vote for Smith and 47% would vote for Jones. The results were stated as being accurate within 3.8 percent points, 19 times out of 20. Who will win the election?

Smith 53%	Jones 47%
$53 \pm 3.8\%$	$47 \pm 3.8\%$
49.2% – 56.8%	43.2% – 50.8%

Can't say for sure, it is unlikely though that Jones would win, 19 times out of 20.

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