5.1: An Introduction to Polynomials

Lesson Goals

- Review the concept of a variable.
- Examine polynomials and classify an expression as monomial, binomial, or trinomial based on the number of terms.
- State the degree of a polynomial.
- Create a model for a given polynomial.

Recap: Elementary Algebra involves variables and the rules for manipulating them.

Definition

A **variable** is a letter that represents an unknown quantity or number.

Examples of variables are: x, n, t, symbols, Greek symbols etc.

Definition

A term is a single number or variable or the product of several numbers or variables.

Examples of terms are: $2xy, 1, 1m^2$.

Definition

A coefficient is a real number multiplied by the variable(s) in a term.

Examples: The coefficient of the term $2xy$ is 2. The term $1m^2$ has coefficient 1.

Definition

An expressions involves operations of one or more numbers and/or variables.

Examples of expressions are $2x + 3z - z$, $4 \times 4 \times 4$, $\sqrt{9}, a^2 - ab + 2$.

We can use the above definitions to start to define polynomials. The building blocks of polynomials are monomials.

Definition

A **monomial** is a single term. When written using a single variable (such as x), it can be written as cx^n, where c is a real number and n *is* a **non-negative integer.**

Examples of a monomial are $5x^3$ *and* $-4ab$.

Definition

A binomial is the sum or difference of two monomials.

Examples of a bi nomial are $q + 3$, $2w^2 - 7w^5$, *and* $a + b$.

Definition

A trinomial is the sum or difference of three monomials.

Examples of trinomials are $6d^2 + 6 - d^9$, $2v^2 + v - 3$, $a^2 + 5ab - 2b^2$.

Definition

A Polynomial is composed of the sum or difference of one or more monomials.

Sources: CEMC Open Courseware

Examples of polynomials are $4r^3 + 7r^2 - r - 9$ *and* $x^2y - 2xy^2 + 5xy + 2x - 8y + 7$

The Constant Term

In a term cx^n, if $n = 0$, we have

$$cx^n = cx^0 = c \times 1 = c$$

In the case where $n = 0$, c is referred to as the **constant term**, a numerical term with no variable part.

Example 1

Determine whether or not each of the following is a polynomial. State the constant term.

$5n^4 + 3n^2 + (-1)$

a. $5n^4 + 3n^2 - 1$ — *Polynomial?* Yes $c =$ -1

b. $-3x^2 + x^{-1} + 14x - 20$ — *Polynomial?* No $c =$ -20

c. $3m^1 + 0$ — *Polynomial?* Yes $c =$ 0

d. $8k^{(1)} - 3k^7 + 0$ — *Polynomial?* Yes $c =$ 0

$\sqrt{x} = x^{\frac{1}{2}}$

e. $4\sqrt{x} + 7x$ — *Polynomial?* No $c =$ —

f. $8c - 3(7^c) + 5$ — *Polynomial?* No $c =$ —

$\sqrt{5} \cdot x^1$

g. $x^1 + 5y^1 - 2$ — *Polynomial?* Yes $c =$ -2

c^7

> **Definition**
>
> The degree of a monomial having a single variable is the value of the exponent of the variable. The degree of a monomial having more than one variable is the sum of the exponents of those variables.

Examples: The monomial $2n^4$ *has degree* 4. *The monomial* mn^2 *has degree*_____.

$m^1 n^2$

degree $= 1 + 2$

Definition

The degree of a polynomial is the highest degree of its monomials with non-zero coefficients.

For example, the polynomial $6x^7 - x^4 - x$ has degree 7.

Example 2

State the degrees of the following polynomials. State the coefficient of the first term. Classify them as either monomial, binomial, trinomial, or polynomial. State the variable(s).

a. $4x^7 + 9x^6 + 3x^2 - 11$

b. $-3x^5 - 4x^6 - 10x^{12}$

c. $-13x^{15} - 8x^9 + 7x + 2$

d. $30u^{14}v^{16} + 60u^{28}v^{28} - 1$

e. $2xy^2 + 5$

f. $3r$

g. $4r^0$

	Degree	*Coefficient of first term*	*Classification*	*Variable(s)*
a.	7	4	polynomials	x
b.	12	-3	trinomials	x
c.	15	-13	polynomial	x
d.	56	30	trinomial	u, v
e.	3	2	binomials	x, y
f.	1	3	monomials	r
g.	0	4	monomials	none

Sources: CEMC Open Courseware

Representing Polynomials with Algebra Tiles

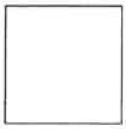
The absolute value of a tile represents its area. The color of a tile indicates its sign. For example, a 1x1 tile is represented by a square of side length 1 and area of 1 square units. The shaded color is positive (+1), and the white color is negative (-1). In your textbooks, usually +1 is represented as red, and -1 is represented by white.

 *+1 tile*  *−1 tile*

For example, an $x \times 1$ tile is represented by a rectangle with length x and width 1 with an area of x square units. The black color is positive $(+x)$ and the white color is negative $(-x)$. In your textbooks, these are usually green and white, respectively.

 *+x tile*  *−x tile*

For example, an $x \times x$ tile is represented by a square of side length x and area x^2 square units. The black color is positive $(+x)$ and the white color is negative $(-x)$. In your textbooks, these are usually green and white, respectively.

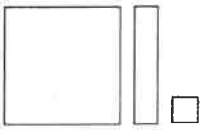
 x^2 *tile*  $-x^2$ *tile*

Example 3

Write an expression for each set of algebra tiles

a) 

$2x + 4$

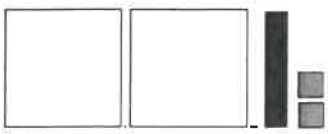
b) 

$-x^2 + (-x) + (-1)$

$-x^2 - x - 1$

c) 

$3x^2 + 3x - 3$

d) 

$-2x^2 + x + 2$

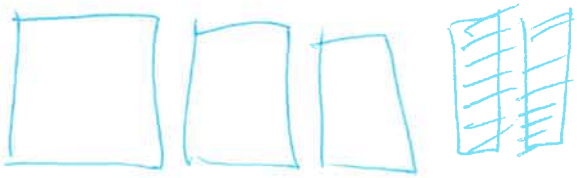
Sources: CEMC Open Courseware

Example 4

TRY

Model each polynomial using algebra tiles.

a. $-3x^2 + 2x$



Hmwk: 179 - 182

13, 11, 21, 23, 24, 28

QUIZ TOMORROW

b. $x^2 + 4$

c. $-1 + 2x^2 + 3x$

Where Will Campers Sleep in 20 Years ?

Do each exercise below, following the directions given for each section. Select your answer from the two choices given and circle the letter next to it. Write this letter in the box at the bottom of the page that contains the number of that exercise.

Write each expression in exponential form.

(1)	$x \cdot x \cdot x \cdot x$	(A) x^3	(E) x^4
(2)	k cubed	(N) k^3	(R) k^6
(3)	$12 \cdot m \cdot n \cdot n$	(M) $12mn$	(S) $12mn^2$
(4)	$\frac{1}{3} \cdot u \cdot u \cdot u \cdot v \cdot v$	(O) uv^2	(T) $\frac{1}{3}u^3v^2$
(5)	$(a+b)(a+b)(a+b)$	(E) $(a+b)^3$	(L) a^3+b^3
(6)	$(c+d)(c+d)(c-d)$	(A) $(c-d)^3$	(U) $(c+d)^2(c-d)$
(7)	$-7 \cdot x \cdot (x+3)(x+3)$	(A) $-21x^3$	(R) $-7x(x+3)^2$
(8)	$(x+y)$ squared	(E) $(x+y)^2$	(I) x^2+y^2
(9)	the fifth power of the product of p and q	(I) $(pq)^5$	(O) $(p+5)q$

Evaluate each expression for the given values of the variables.

(10)	x^2-3xy	if $x=5$, $y=2$	(T) -5	(R) 10
(11)	x^2-y^2	if $x=-7$, $y=-1$	(H) 48	(R) 52
(12)	$(x-y)^3$	if $x=2$, $y=-4$	(A) 256	(U) 216
(13)	xy^2-2x^3	if $x=3$, $y=2$	(F) -42	(P) -56
(14)	$\frac{-5a^2}{a-b}$	if $a=-4$, $b=6$	(R) 12	(T) 8
(15)	$\frac{3ab^3}{(2a)^2}$	if $a=1$, $b=-2$	(P) 4	(T) -6
(16)	$\frac{(a+b)^4}{9-a^2}$	if $a=-5$, $b=3$	(R) 2	(N) -1

9	2	4	11	1	13	6	15	12	7	5	14	8	16	10	3